A random triadic process

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joint work with Yuval Peled and Benny Sudakov

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Will the result have large independent sets?

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We consider a "triadic process" that adds

- an edge not closing any triangle with probability 0
- an edge in a triangle with probability p, independently for each triangle.

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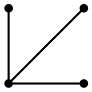

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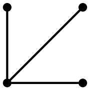

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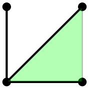

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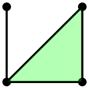

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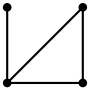

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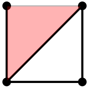

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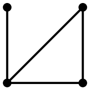

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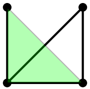

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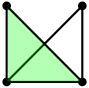

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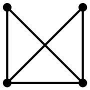

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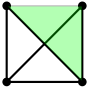

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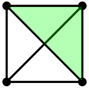

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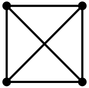

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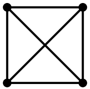

Definition

We say the process *propagates* if $G = K_n$ at the end.

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- *If* $p \leq \frac{1-\varepsilon}{2\sqrt{n}}$ *then the triadic process does not propagate whp: at the end* G *only has* $O(n\sqrt{n})$ *edges.*

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- *If* $p < n^{-\frac{1}{2}-\varepsilon}$ *then* $\pi_1(Y(n, p)) \neq 0$ *whp.*
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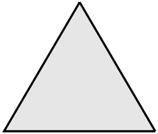

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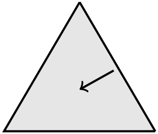

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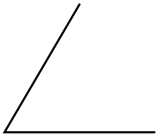

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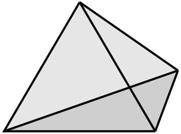

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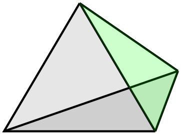

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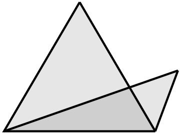

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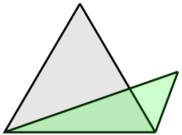

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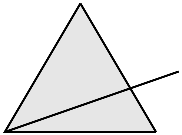

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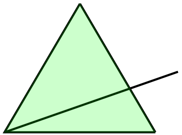

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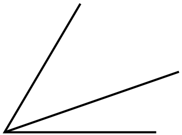

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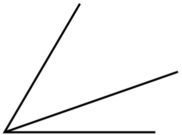

The inverse of a collapse sequence is a propagating triadic process!

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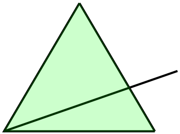

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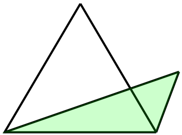

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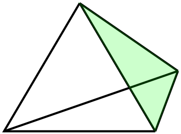

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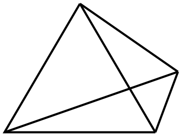

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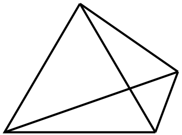

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- If $p \geq \frac{1+\varepsilon}{2\sqrt{n}}$ then the triadic process propagates on $H(n, p)$.

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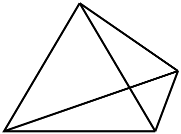

The inverse of a collapse sequence is a propagating triadic process!

- If $p \geq \frac{1+\varepsilon}{2\sqrt{n}}$ then the triadic process propagates on $H(n, p)$.
- The inverse process is a collapse sequence in $Y(n, p)$.

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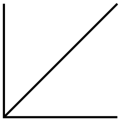

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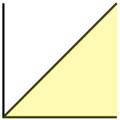

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- Pick an open triple uniformly at random and sample it

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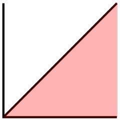

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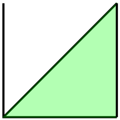

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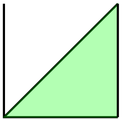

Our process

- Pick an open triple uniformly at random and sample it
- If it is in H, add the missing edge to G
- repeat

The differential equation method

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Tool for tracking variables in a random process.

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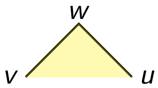

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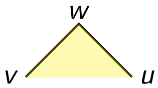
open triple at v: 

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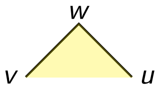

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By symmetry, $D_v(i) \approx D(i)$ and $F_v(i) \approx F(i)$

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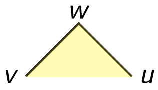

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By symmetry, $D_v(i) \approx D(i)$ and $F_v(i) \approx F(i)$

Observation

Total number of open triples $\approx \frac{nF(i)}{2}$

Tracking the degrees

How do the degrees change in one step?

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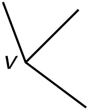

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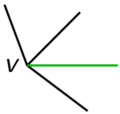

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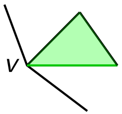

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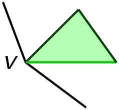

$$D_v(i+1) - D_v(i) = \begin{cases} 1 & \text{if an open triple at } v \text{ is successfully sampled} \\ & \end{cases}$$

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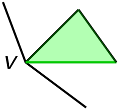

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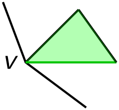

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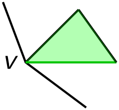

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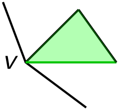

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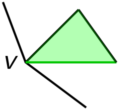

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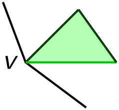

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$$\boxed{d'(t) = 2c}$$

where $p = \frac{c}{\sqrt{n}}$.

Tracking the number of open triples

There are two ways to gain:

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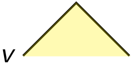

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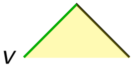

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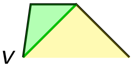

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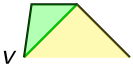

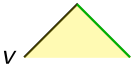

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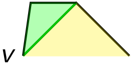

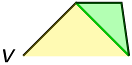

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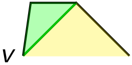
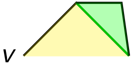


$\approx F(i)D(i)$
possibilities



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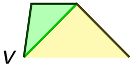

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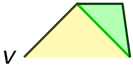

We lose an open triple when we sample it: $F(i)$ options

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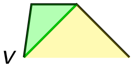

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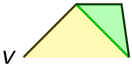

We lose an open triple when we sample it: $F(i)$ options

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Tracking the number of open triples

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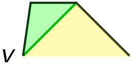

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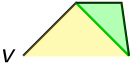

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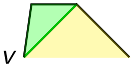

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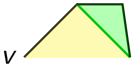

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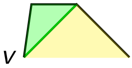

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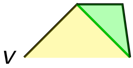

We lose an open triple when we sample it: $F(i)$ options

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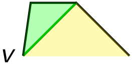

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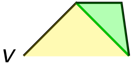

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$$\boxed{f'(t) = 4cd(t) - 2}$$

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$$d'(t) = 2c \qquad f'(t) = 2cd(t) - 2$$

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Number of open triples

Trajectories

$$\begin{aligned} d'(t) &= 2c & f'(t) &= 2cd(t) - 2 \\ d(0) &= 0 & f(0) &= 1 \end{aligned} \qquad \text{(star)}$$

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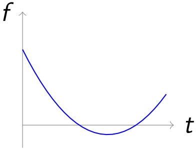
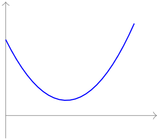


$c < \frac{1}{2}$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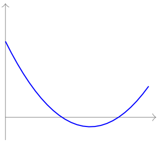
Endgame



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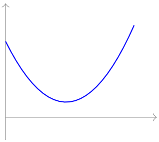

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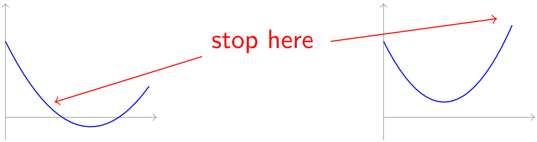

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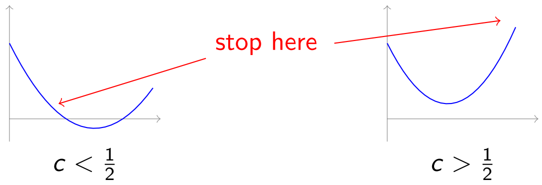
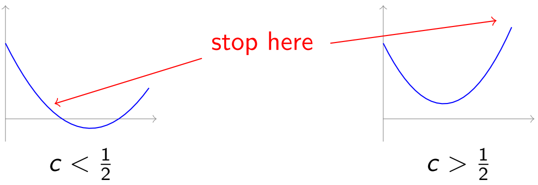


- sample in rounds: all available open triples simultaneously

Endgame



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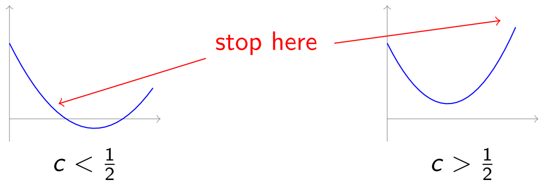

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When $c < 1/2$:

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When $c > 1/2$:

- degrees and open triples grow exponentially

A slight issue

To obtain the approximate formula for $F(i+1) - F(i)$, we needed the codegrees to be small.

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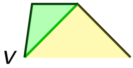

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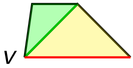

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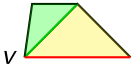

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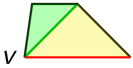

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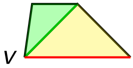

- The codegrees are not concentrated
- No easy way to estimate them
- We need to track two more variables

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   - seems unlikely..
2. What if we do not fix the tree?
   - Given $H(n, p)$, $p = \frac{1-\varepsilon}{2\sqrt{n}}$, is there a tree that the process propagates from?

Thank you!